

## An Ad Hoc Method for Computing Pseudo-Effect Size for Mixed Models

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### ABSTRACT

Effect size is increasingly being reported in journals across multiple domains. Many fields that use complex experimental designs, such as psychology and engineering, must analyze those designs using mixed models. Because of the intrinsic nature of mixed model computations, none of the packages (R, SPSS, SAS®, STATA, etc.) provide effect sizes. Hence, when analyzing models that use fixed and random factors, such as split-plot and repeated measures designs, an alternative method must be used to compute effect size using output from procedures capable of appropriately analyzing mixed models. This paper presents an ad hoc method of computing effect size by working backwards from the F-values output by SAS® PROC MIXED to produce eta-squared, omega-squared, and partial eta-squared pseudo estimates. A SAS® Macro will be provided to perform the calculations.

*Keywords: Effect Size, Mixed Model*

### INTRODUCTION

SAS® PROC MIXED provides a flexible environment for the construction of mixed models for the analysis of experiments, allowing the user to adjust for random or repeated variables, different covariance structures, and unbalanced datasets (Jones & Huddleston, 2009). However, PROC MIXED uses Reduced Maximum Likelihood (REML) or Maximum Likelihood (ML) to estimate parameters. As a result, there is no ANOVA table and thus there are no effect size estimates.

### Importance of Calculating Effect Size

Since 1999 the American Psychological Association (APA) began strongly recommending that effect sizes and interval estimates be presented for primary experimental outcomes, a call that was more strongly relayed in their 2010 Publication Manual (Fritz, Morris, & Richler, 2012; Kelley & Preacher, 2012). These APA standards are commonly used across multiple fields, particularly in the social sciences. Effect size measures are a standardized index whose goal is to estimate a parameter independent of the sample size (Olejnik & Algina, 2003), thus quantifying the relationship between experimental variables and dependent measures in a more robust manner than  $p$ -values. The following briefly discusses the computation and interpretation of three of the most common effect size measures that represent the proportion of variability explained by a particular factor: eta-squared ( $\eta^2$ ), omega-squared ( $\omega^2$ ), and partial-eta squared ( $\eta_p^2$ ).

Eta-squared ( $\eta^2$ ) and omega-squared ( $\omega^2$ ) have much in common. Eta-squared ( $\eta^2$ ) (Equation 1) is the ratio of variability for an experimental variable compared against all other forms of variability in the analysis (Fritz, Morris, & Richler, 2012). Omega-squared ( $\omega^2$ ) (Equation 2) similarly looks at this ratio, except on the population level and is thus less bias than  $\eta^2$ .

$$\eta^2 = \frac{SS_{effect}}{SS_{total}} \quad (\text{Equation 1})$$

$$\omega^2 = \frac{SS_{effect} - (df_{effect} \times MS_{error})}{SS_{total} + MS_{error}} \quad (\text{Equation 2})$$

Partial eta-squared ( $\eta_p^2$ ) (Equation 3) is the ratio of variability of the dependent measure explained by the experimental variable when controlling for other experimental variables (Fritz, Morris, & Richler, 2012):

$$\eta_p^2 = \frac{SS_{effect}}{SS_{effect} + SS_{error}} \quad (\text{Equation 3})$$

$\eta_p^2$  builds off its original  $\eta^2$  counterpart by adjusting the denominator from being all forms of variability in the model (i.e.,  $SS_{total}$ ) to being just the amount of unexplained variation in the dependent measure (i.e.,  $SS_{error}$ ) plus the amount of variation explained by the experimental variable (i.e.,  $SS_{effect}$ ). This change allows experimenters to compare effect sizes across multiple experiments when studies have the similar designs that result in comparable error terms (Lakens, 2013; Olejnik & Algina, 2003).

In experimental psychology,  $\eta_p^2$  is commonly the most frequently reported effect size as it is automatically produced when running a GLM model in SPSS (Fritz, Morris, & Richler, 2012). However,  $\eta_p^2$  is neither automatically produced when using mixed models nor always the most appropriate effect size measure to report in all instances.

### Challenges in Computing Effect Size for Mixed Models

PROC MIXED allows the user to fit various forms of mixed linear models to make statistical inferences. A mixed linear model can be written using the following setup:

$$Y = X\beta + Z\gamma + \varepsilon \text{ where } \gamma \sim N(0, G), \varepsilon \sim N(0, R) \\ V(Y) = Z^t G Z + R$$

$$\text{Test } H_0: L^t \beta = 0, F = \frac{\hat{\beta}^t L [L \hat{C} L^t]^{-1} L^t \hat{\beta}}{\text{rank}(L)}$$

This setup shows how mixed linear models are a generalization of the standard linear model used in GLM procedures (Jones & Huddleston, 2009). Hence, in cases when a standard linear model is evaluated using PROC MIXED, this will result in the same output as would using a GLM procedure. However, PROC MIXED is more malleable than GLM procedures as  $\beta$  is no longer a function of linear data. In fact, mixed model solutions depend on covariance parameter estimates, and multiple options exist for editing the structure of the covariance matrix in PROC MIXED (Jones & Huddleston, 2009; Littell, Stroup, Milliken, Wolfinger, & Schabenberger, 2006).

PROC MIXED's use of REML to estimate variance parameters is often preferred to ANOVA estimates in circumstances when the experimental design includes RANDOM or REPEATED factors that impact those values. This is because PROC MIXED allows the user to alter the model's covariance structure in specific ways that allow the user to appropriately

analyze split-plot designs (Littell et al., 2006). More specifically, the RANDOM statement allows the user to specify random terms in the model, giving them the option to choose the covariance structure of the random effects and to specify hierarchies of effects (note that more than one RANDOM statement may be used in a model) (Moser, 2004). Similarly, the REPEATED statement allows the user to choose the covariance structure, to specify the subject to which repeated measurements belong to, and to use of different structural parameters on different group levels (note that only one REPEATED statement may be used in the model) (Moser, 2004).

### An Ad-Hoc Approach to Computing Effect Sizes from PROC MIXED

While GLM procedures produce an ANOVA table (e.g., Table 1) that can be used to compute effect sizes, PROC MIXED does not.

Table 1. Classical ANOVA table for a within-subjects design.

Source	Sum of Squares (SS)	Degrees of Freedom (df)	Mean Square (MS)	F-values
Treatment	$SS_{effect}$	$(k - 1)$	$MS_{effect}$	$\frac{MS_{effect}}{MS_{error}}$
Subject	$SS_{subject}$	$(n - 1)$	$MS_{subject}$	$\frac{MS_{subject}}{MS_{error}}$
Error	$SS_{error}$	$(k - 1)(n - 1)$	$MS_{error}$	
Total	$SS_{total}$	$(N - 1)$		

However, as the  $F$ -value produced through PROC MIXED is still representative of the ratio of the sum of squares of the effect to the sum of squares error (e.g.,  $\frac{MS_{effect}}{MS_{error}} = \frac{SS_{effect}/df_{effect}}{SS_{error}/df_{error}}$ ), this value can be used as a basis to compute an ad-hoc estimate of ANOVA table values, which can then be used to calculate a pseudo-effect size. Given  $F_{effect} \cong \frac{MS_{effect}}{MS_{error}}$ , we can obtain  $SS_{effect} = df_{effect} \times F_{effect} \times MS_{error}$ . More specifically, for this Macro, the variance component of the  $MS_{error}$  equation was computed by using predictive residuals obtained through the *outpm* statement. This relationship between  $F_{effect}$  and the sums of squares makes it possible to compute pseudo-effect size approximations based on  $F_{effect}$ .

### CASE STUDIES

The following two examples were taken from the Mixed Models Analyses Using SAS® (2012) and explore the calculation of pseudo-effect size based on output from PROC MIXED. The first case includes a REPEATED term and the second case includes a RANDOM term. Code for a SAS Macro to perform the computation is provided in the APPENDIX. Note that the user must choose and enter a REPEATED statement, a RANDOM statement, or both statements into the Macro in order to complete the code.

### Case 1. PROC MIXED with REPEATED term (no RANDOM effect)

Consider a pharmaceutical company that wants to examine the effects of three drugs, drugs *a*, *c*, and *p*, on the respiratory ability of asthma patients:

- Drug *a* is a standard asthma therapy;
- Drug *c* is a new therapy the company developed that is being tested; and
- Drug *p* is a placebo.

Each of the three drugs was randomly assigned and administered to 24 patients, resulting in a total of 72 patients enrolled in the study. Immediately prior to treatment, a baseline *fev1* (i.e., *basefev1*) (forced exhaled volume in one second) was collected; post treatment, *fev1* was collected every hour for eight hours. Data from this study were stored in the SAS datasets, **aglm.fev1uni**. Table 2 shows the resulting calculations for effect sizes using PROC MIXED with the REPEATED statement.

Table 2. *Effect size estimates for fev1 using PROC MIXED with the REPEATED statement.*

	$\eta^2$	$\omega^2$	$\eta_p^2$
Drug	.013	.012	.029
Hour	.040	.034	.082
Drug*Hour	.026	.016	.057

### Case 2: PROC MIXED with RANDOM term (no REPEATED effect)

Consider an agricultural researcher that wants to examine the effects of three seed growth methods (i.e., three fixed factor levels) among five varieties of turf grass that were chosen randomly from the entire population of turf grasses (i.e., five random factor levels). The researcher plants six pots for each method-by-variety combination (i.e., 15 treatment combinations which form a random interaction effect), resulting in a total of 90 pots, all of which are placed in a uniform growth chamber, and forming a completely randomized experimental design. The dependent measure is dry matter *yields* that are measured from grass clippings at the end of a four week period. Data from this study were stored in the SAS dataset, **aglm.grass**.

Because both random and fixed effects are involved, the model is defined as being mixed. The varieties are random. Inferences about the method suggest that differences should apply across all varieties. Table 3 shows the resulting calculations for effect sizes using PROC MIXED with the RANDOM statement.

Table 3. *Effect size estimates for yield using PROC MIXED with the RANDOM statement.*

	$\eta^2$	$\omega^2$	$\eta_p^2$
Method	.149	.133	.183

## DISCUSSION

Computing effect sizes for mixed models is becoming increasingly important across multiple domains. However, no existing code within the SAS Proc Mixed function allows for the direct computation of effect size measures. The Macro presented here seeks to fill this gap by providing researchers with a method to compute pseudo-effect size.

The primary limitation of using this ad-hoc method is that the effect sizes produced should only be compared with other effect sizes produced using the same method. Simulation

research is needed to determine standard values about whether a particular effect size is small, medium, or large, within a model.

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## APPENDIX: Macro Code

```
%macro effectsize_rep(file=,y=,class=, fixed = );
```

```
TITLE1 "Mixed Models Analysis of: &Y";
```

```
PROC MIXED DATA = &file  
          PLOTS(ONLY)=ALL  
          METHOD=REML;  
          CLASS &class;  
          MODEL &y =&fixed / outpm=outpm1 solution  
          ddfm=kr;
```

```
** NOTE: Choose REPEATED, RANDOM, or both statements for YOUR model  
and enter here;
```

```
* For REPEATED the user must enter the correct REPEATED statement and  
RERUN the Macro;
```

```
e.g., repeated /subject=patient(drug) type=ar(1) R ;
```

```
* For RANDOM the user must enter the correct RANDOM statement and  
RERUN the Macro;
```

```
e.g., random variety method*variety / ;
```

```

ods output tests3=tests3;
RUN; QUIT;

%_eg_conditional_dropds(var2);

PROC MEANS DATA=WORK.outpml nonobs noprint
  FW=12
  PRINTALLTYPES
  CHARTYPE
  VARDEF=N
    VAR
      N ;
  VAR &y Resid;

OUTPUT      OUT=var3
            VAR()=
            N()=
  / ;
RUN;

data var4;
set var3;
id=_n_;
run;

data test_eta;
set tests3;
id=_n_;
Run;

%_eg_conditional_dropds(SASUSER.QUERY_FOR_TEST_ETA);
%_eg_conditional_dropds(SASUSER.test_eta_2);

PROC SQL;
  CREATE TABLE SASUSER.test_eta_2 AS
  SELECT t1.*,
         t2.*

    FROM WORK.TEST_ETA t1
         , WORK.VAR4 t2 ;
  quit;

data sasuser.&y;
set sasuser.test_eta_2;

mse =resid*(_freq_-1)/_freq_;

ss_effect = numdf*Fvalue*mse;
ss_total = (_freq_-1)*&y;
ss_error = mse*(_freq_-numdf);

```

```
eta_2=ss_effect/ss_total;  
omega_2 = (ss_effect-(numdf*mse))/(ss_total+mse);  
partial_eta_2=ss_effect/(ss_effect+ss_error);  
  
run;  
  
%mend;
```